MINT: PROPOSITION OF A NEW TRANSIT ASSIGNMENT ALGORITHM FOR FREQUENCY BASED NETWORKS

PALMIER Patrick

CETE Nord Picardie / Ministère des Transports, FR

1 INTRODUCTION

As a public service of engineering and a member of the scientific and technical network of the French Ministry for Sustainable Development, CETE Nord Picardie conducts studies for state departments and local authorities.

With a long experience in transport modeling and planning, it has now become a specialist in mobility data collection and analysis.

In public transport modeling and demand assignment, the optimal strategy method is often used for transit assignment. The objectives of this paper are to present the new Mint method, based on the optimal strategy method but overcoming some of its limitations and to analyze implicit assumptions on time tables on which theses two methods are based.

2 BACKGROUND

The concept of optimal strategies was introduced by Spiess - Florian in 1989.

The model is based on the assumption that a user of public transport, who have a range of attractive strategies to get to their destination, will board in the first vehicle of an attractive line that comes to him.

This algorithm is designed for frequency based transit networks. It was first implemented in the original INRO software EMME / 2, and is currently available in various other modeling softwares.

The implementation of the algorithm consists of two steps:

- Start from the destination and identify attractive lines to get to this destination.
- Then distribute the demand, starting from the origin, and successively until the destination, in proportion to the frequency of attractive lines, starting from each node.

3 LIMITS

Several limitations in the standard implementation of the algorithm are identified :

- L1 : The model is only based on frequency and doesn't take into account travel times. As a consequence, if a change in travel times of one or several lines does not impact the set of attractive line for a specific origin-destination,the distribution of flows between these lines remains unchanged. This is very restricting because it keeps from evaluating the effect of a improvement in the commercial speed of a line, such as the introduction of a bus rapid transit.
- L2 : Pedestrian links are considered as transit lines with an infinite frequency. Therefore, if in a node, both a set of lines and a walking link are attractive, the traffic will be fully assigned on the walking link. So, traffic distribution depends on how the network is described, especially on transfer nodes. For example, the two following descriptions of the same real network lead to different results: a single node where all lines converge, and two nodes linked by a small (or even fictive) pedestrian link and with lines split between the two nodes. In the second case, demand is only assigned on the attractive lines of a single node.
- L3 : Similarly, once a line has been chosen, the frequency of the downstream segment of this line is considered infinite, because the user is already in the vehicle. So, if another line is attractive on the next node, the user will remain in the vehicle even if the interchange saves time.

For these reasons, INRO has recently introduced in the latest version of Emme a transit assignment with variant procedures overcoming some of these limitations.

In the following, we propose another algorithm base on a new approach called Mint, overcoming these limitations.

4 OBJECTIVE OF THE MINT ALGORITHM

The aim of this new algorithm is, while keeping the basic principles of optimal strategies, to propose an adaptation to overcome the above limits:

- to take into account the travel time in the distribution of transit volumes between the attractive lines (limit L1),
- to propose a distribution of volumes according to any attractive line, including transit lines and pedestrian links. (limit L2),
- to be able to provide a flow distribution between the line in which the user travel and another attractive line that starts at a later node(limit L3)
- to manage routes, auxiliary transit only , which do not focus all the demand
- to ensure continuity in the distribution of volumes functions, depending on frequencies and travel times.
- to improve the convergence of an iterative algorithm, which takes into account the capacity constraints.

4.1 Principles

- If a line is attractive, a fraction of the corresponding demand must be assigned to the line.
- If the travel time of a line is improved, even if the set of lines attractive does not change, an additional fraction of demand must be assigned to the line.
- The basis of optimal strategies must be maintained, without introducing additional parameter nor variable nor method of selection of additional routes

4.2 First computation elements:

5 ALGORITHM DESCRIPTION

Notations:

- t : travel time
- hdw : headway
- v : proportion of flow assigned to the line

5.1 Example 1

- Optimal strategies

Both lines are attractive. The distribution of flows is defined in proportion to the frequency (5 of 7 vehicles for line 1, 2 of 7 vehicles for line 2)

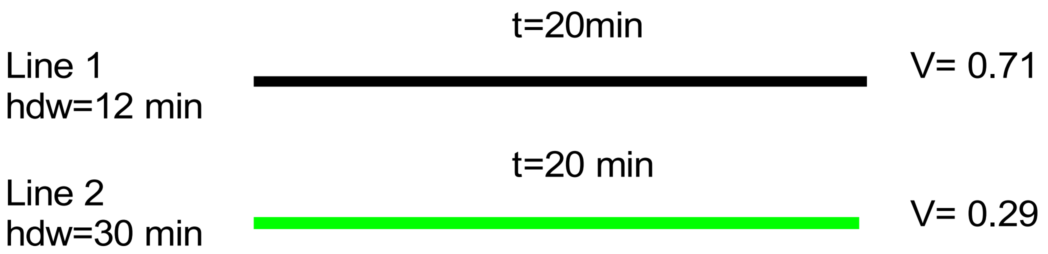


Figure 1: optimal strategies, same travel times

- Mint procedure

One of the main adaptations of optimal strategies concerns the distribution between attractive lines, that does not occur in proportion to the frequency.
We need to compute combined frequencies differently

- Step 1: Combined frequencies calculation

Total travel time on line 1 varies from 20 and 32 minutes

Total travel time on line 2 varies from 20 and 50 minutes where Total travel time= travel time + waiting time

Then, we calculate the MINT (MINimum maximum Time) : 32 minutes (minimum of 32 and 50)

Principle: If the expected travel time using the line 2 is greater than 32 minutes, users will prefer to use the line 1 that guarantees a time of 32 minutes maximum

Line 1 : The sum of the attractive waiting times is 60 minutes (32-20)x5=12x5 vehicles

Line 2 : The sum of the attractive waiting times is 24 minutes (32-20)x2=12x2 vehicle arrivals, because if the expected travel time using line 2 is greater than 32 minutes, passengers will choose the line 1

The waiting times for lines 1 and 2 taken independently, lead to a total of attractive waiting time of 84 minutes in an hour corresponding to 7 vehicles arrivals. By combining both lines, the sum of the attractive waiting time in an hour must be 60 minutes. We had to deal with an extra wait time of 24 minutes. The Mint main hypothesis consist in saving the same amount of waiting time for each inter-vehicle arrival, that is 24 / 7 = 3.43 minutes by vehicle

The expected total travel time varies by combining both lines, between 20 minutes and 28.57 minutes (28.57 = 32 – 3.43)

- Step 2: computing flows

In Mint, contributions of each line are calculated as follows:

$$p_i = \frac{M - t_i}{hdw_i}$$

$M : Minimum\ maximum\ time\,(MINT)$

$t_i : minimum\ travel\ time\ on\ line\ i$

$hdw_i : headway\ on\ line\ i$

$p_i : volume\ proportion\ on\ line\ i$

As travel times are equal on both lines, the volumes on both lines follow the proportion of the frequency and the result is the same as optimal strategies.

- Step 3 : Expected travel time calculation

$$T = \frac{1}{2} \sum_{i \in S} p_i (m_i + M)$$

$T = total\ expected\ travel\ time$

$p_i = volume\ proportion\ on\ line\ i$

$S = the\ set\ of\ attractive\ lines$

The expected travel time is 0.5*((28.57-20)/12)*(28.57+20)+((28.57-20)/30)*(28.57+20)) 24.285 minutes

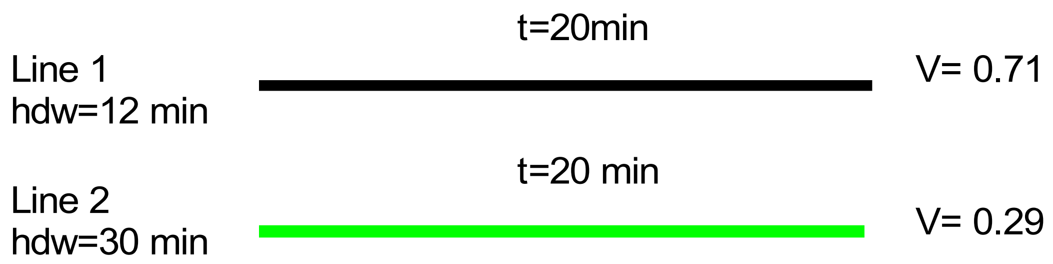


Figure 2: Mint algorithm, same travel time

If line 1 and line 2have the same travel time, both Mint and optimal strategies produce the same results in terms of flows distribution and expected travel time.
But, if travel time is smaller on line 2, it had sense that, compared to the situation in Figure 1, a higher proportion of passengers use the line 2

5.2 Example 2

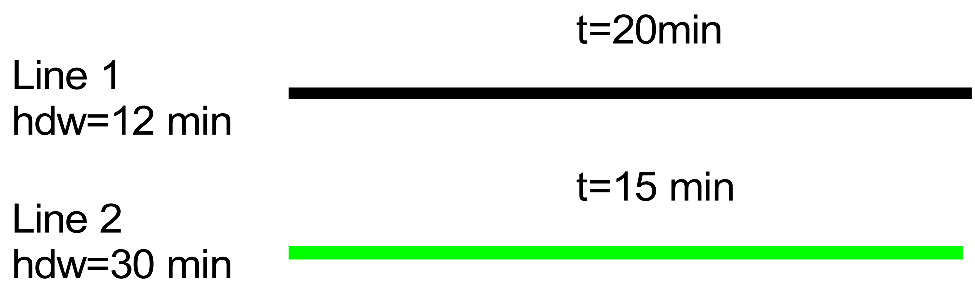


Figure 3: Network example n°2

Considering users with uniform arrivals and perfectly aware of the transit network, their expected travel time is between :

- Line 1: between 20min and 32minutes
- Line 2: between 15min and 45 minutes

The minimum maximum time between both lines is 32minutes (20+12)

- Step 1: combined frequencies calculation

Line1 : total attractive waiting time(32-20)*5=60 minutes for 5 vehicles
Line 2 : total attractive waiting time (32-15)*2=34 minutes for 2 vehicles

So, by combining both lines, the total attractive waiting time in an hour must be of 60 minutes, which allows to save a waiting time of 34 / 7 = 4.86 minutes by bus.

The expected travel time varies in combining both lines between 20 minutes and (32-4.86)=27.14 minutes.

- Step 2: computing flows

line 1=(27.14-20)/12=0.595
line 2=(27.14-15)/30=0.405

- Step 3: expected travel time computation

The average expected travel time: 0.5*((27.14-20)/12)*(27.14+20)+((27.14-15)/30)*(27.14+15)) = 22.56 minutes

The travel time is lower than the one found in optimal strategies and the distribution of flow is also different.

Results:

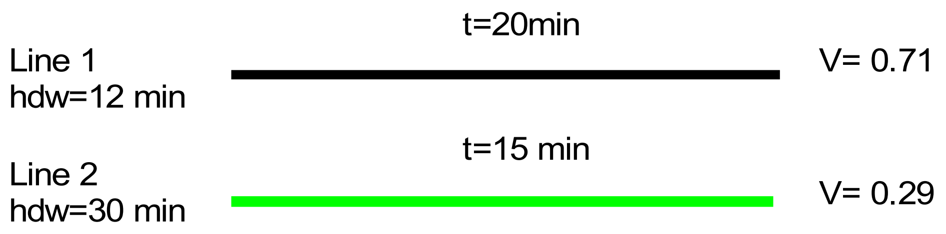


Figure 4: Optimal strategies with equal travel times

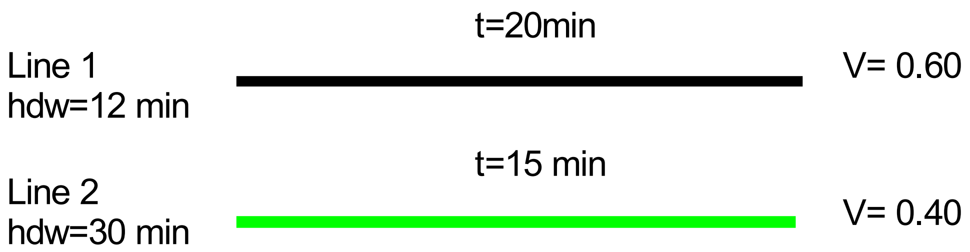
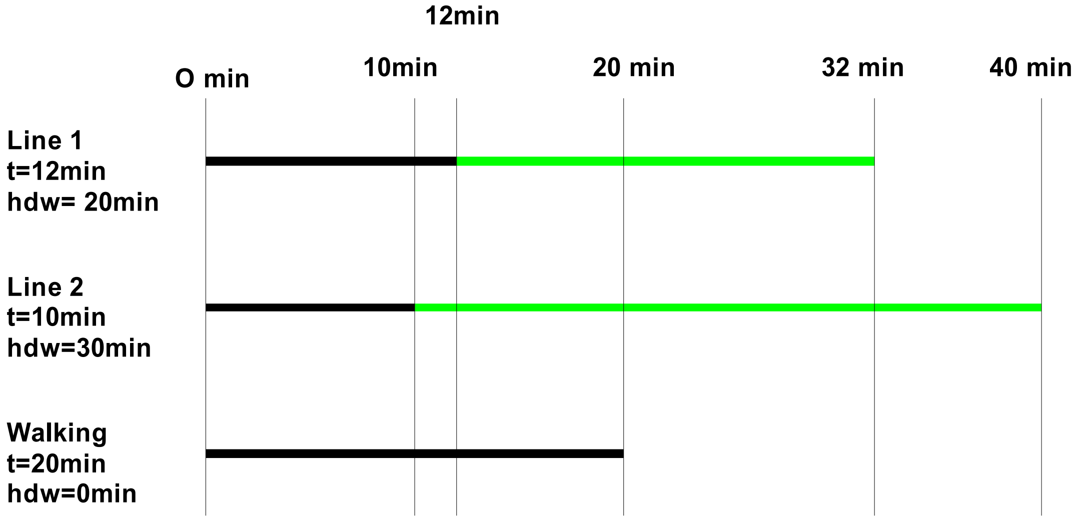


Figure 5: Mint Algorithm with different travel times

In addition, we check the consistency of the model with the optimal strategy model : in the case of equal travel times the assignment only depends on the proportion of frequencies.

5.3 Application on a network of three lines, one of infinite frequency

- Network characteristics

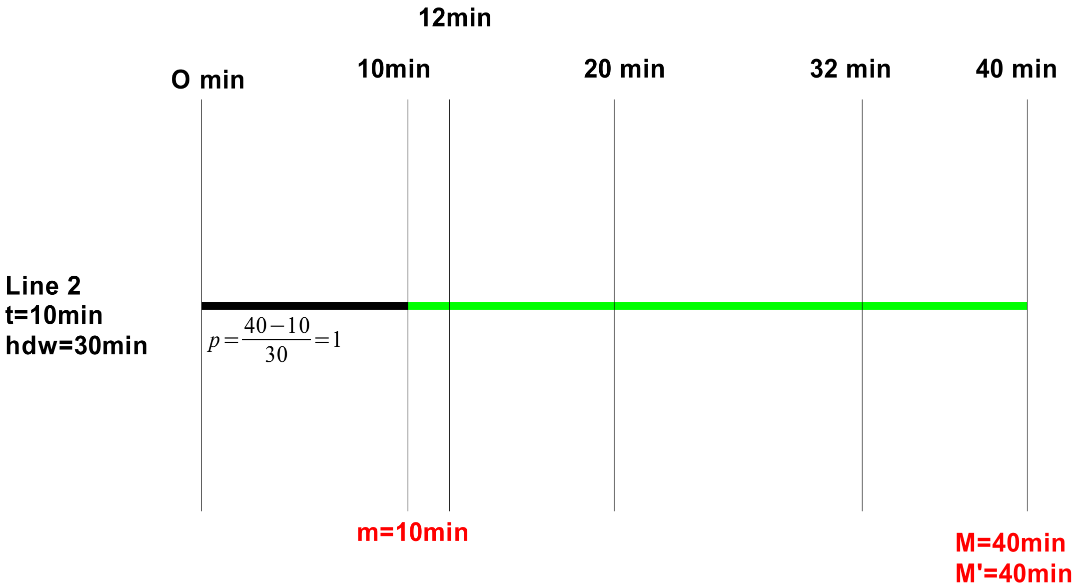


- Definitions

t = travel time
hdw = headway
m = minimum travel time
M = MINT = minimum maximum time.
M' = minimum maximum time excluding walking strategies

- Step 1

We take the line with the smallest travel time

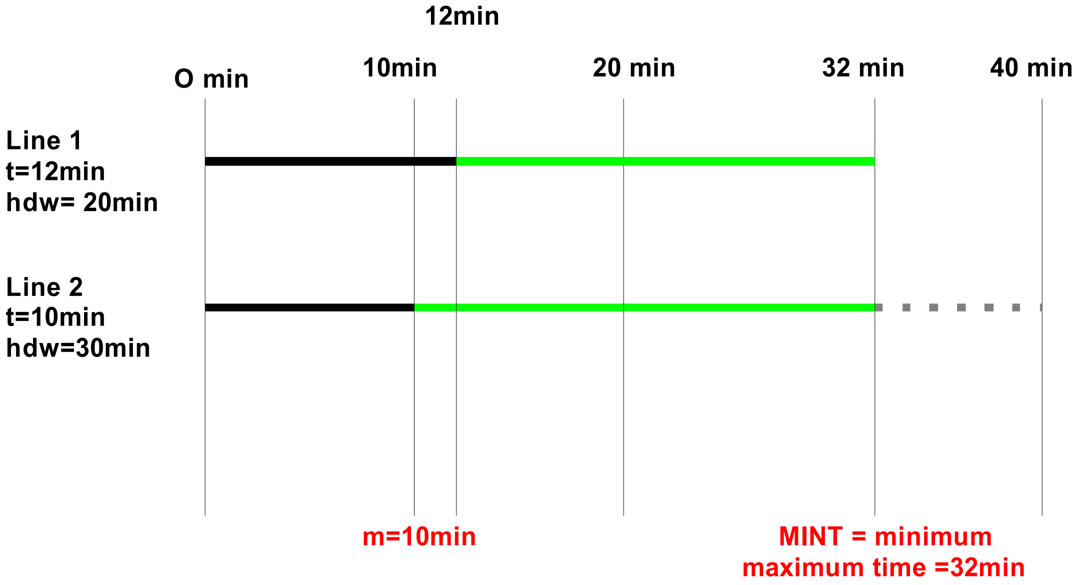


$$T=1.\frac{10+40}{2}=25\,min$$

- Step 2

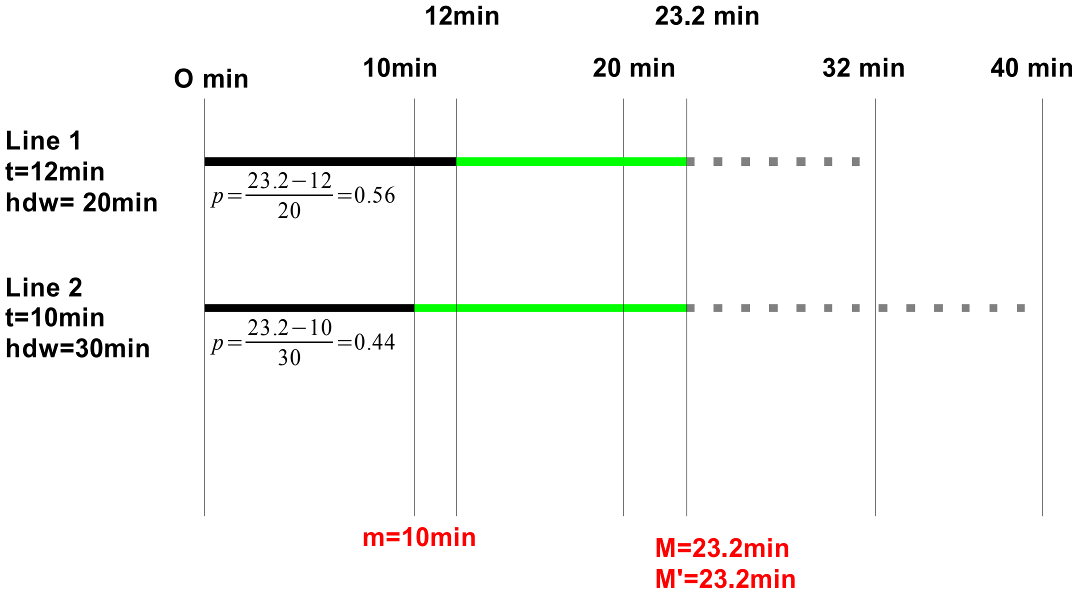
We consider the next line with the smallest travel time :
This is line 1, whose travel time of 12 minutes is lower than the current minimum maximum time (M = 40 min) : line 1 is attractive.

Then, we determine the new minimum maximum time



- Step 3

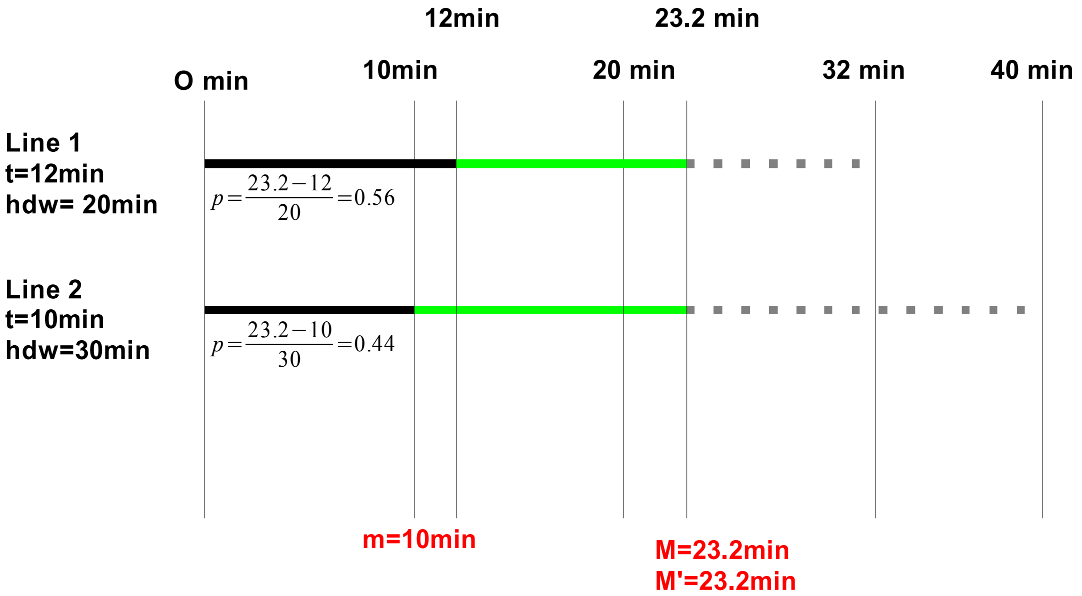
We take into account the combined frequencies of the two lines, reflecting the fact that the sum of the inter-vehicular durations of both lines combined cannot exceed 60 minutes in an hour



$$T=0.56.\frac{10+23.2}{2}+0.44.\frac{12+23.2}{2}=17.04\,min$$

- Step 4

We consider the next line with the smallest travel time : line 3. Its frequency is infinite since it is a walk-only link. The travel time (20 min) is lower than the current minimum maximum time M '(23.2 min). So, the walk-only strategy is attractive



$$T=0.56.\frac{10+23.2}{2}+0.44.\frac{12+23.2}{2}=17.04\,min$$

The infinite frequency property requires special consideration of such a line.

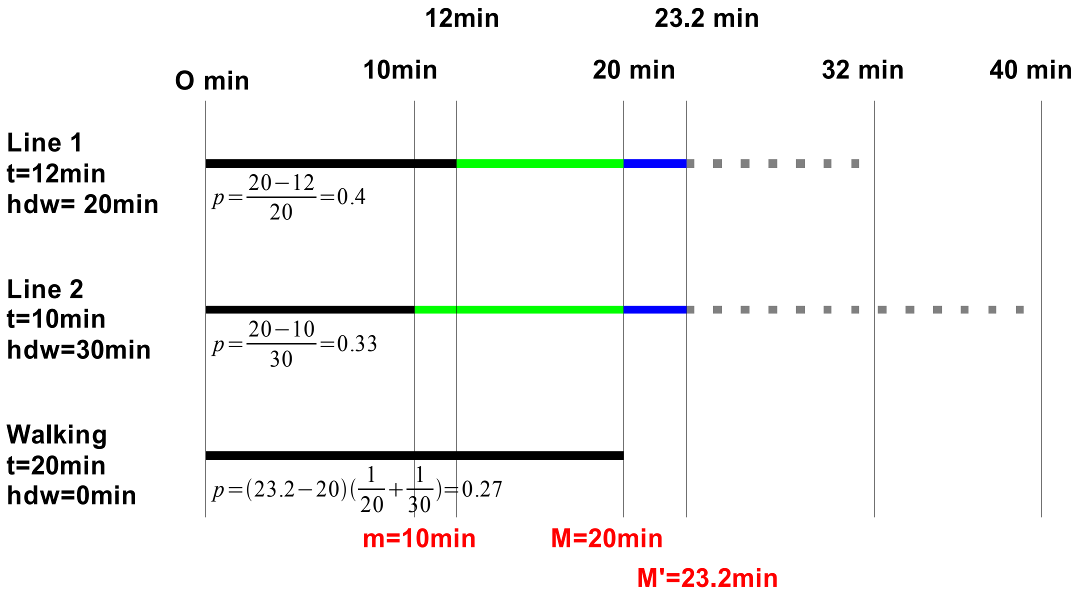
The proportion of volume, for walking strategies are given by the following formulas.

$$p_i=(M'-M)(\sum_{i\in S}\frac{1}{hdw_i})=1-\sum_{i\in S}\frac{M-m_i}{hdw_i}$$

n_i: *number of vehciles per hour for line i*

S: *set of non walking attractive lines*

We deduce the expected total travel time, and distribution according to the different lines.



$$T=0.4.\frac{10+20}{2}+0.33.\frac{12+20}{2}+0.27.\frac{20+20}{2}=16.68\,min$$

6 CONSIDERATION ON WALKING LINKS

6.1 Two kinds of walking links

Walking links can be considered in two ways :

- Type 1: in a specific way.

This method considers that the two strategies below equivalent (excluding weight of time and transfer):

$$L'_1(t=21, Hdw=10)=L_1(t=15, hdw=10)+P(t=6)$$
$$L_{1,}\ L'_1\ are\ transit\ lines\ P\ walk\ link$$

- Type 2: in the same way as a transit line:

This method treats the walking link as a transit line with $hdw=\epsilon$ and $\epsilon \rightarrow 0$

6.2 Concrete example

Lines 1 and 2 have a travel time of 10 minutes, and the length of the walking link is 0

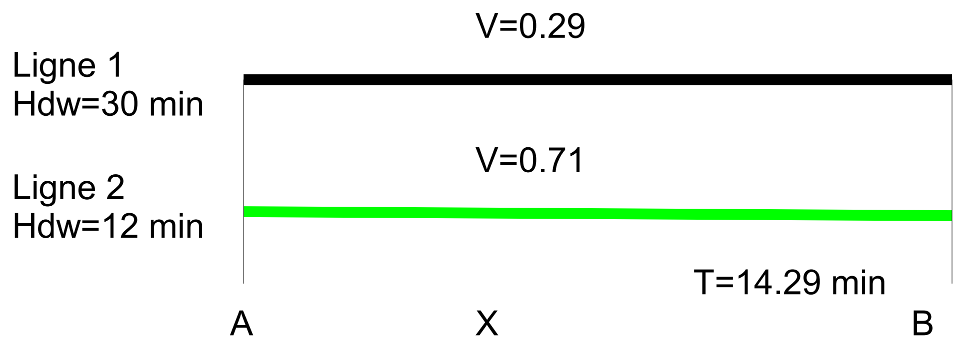


Figure 6: Optimal strategies without walk link

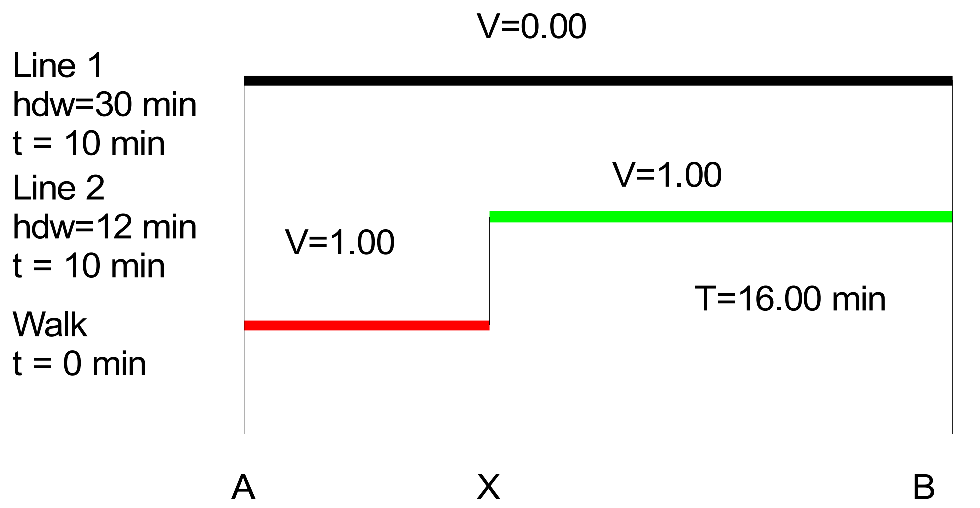


Figure 7: Optimal strategies with walk link

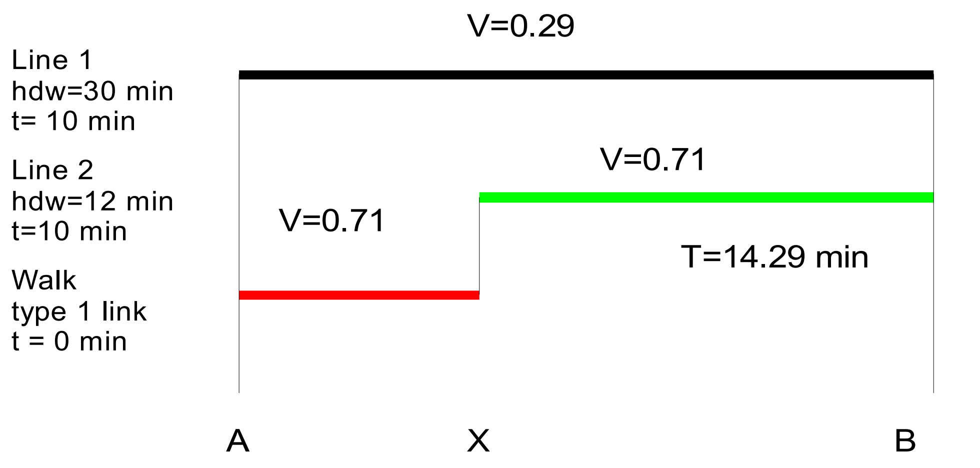


Figure 8: Mint with walk link of type 1

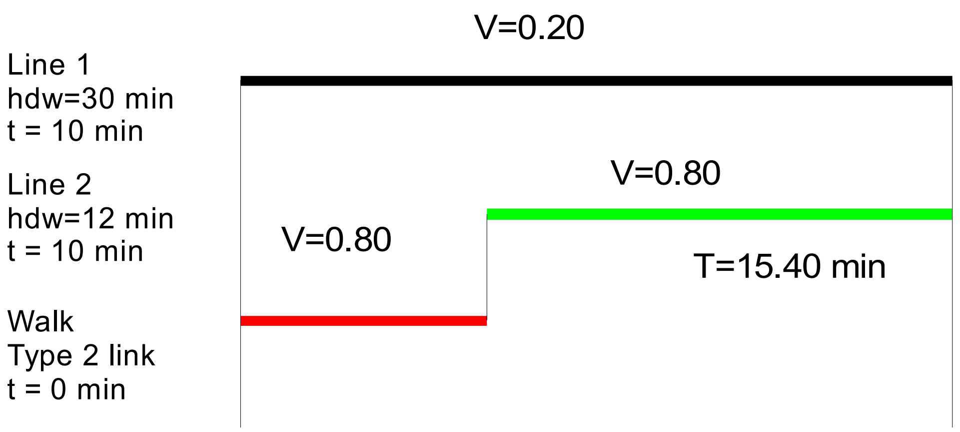


Figure 9: Mint with walk link of type 2

With walking links of type 1, we keep the same travel time and the same distribution of volumes as optimal strategies do, without the walking link. The results of assignment are then independent with the network codification principles.

However, the generalization of this approach for very long pedestrian links seems problematic : it is based on the assumption that the user is very familiar with the timetables of all the lines, and it can access to the the various stops, by integrating the walking time walking in the lines timetables in the neighbourhood.

Mint can take into account the two types of pedestrian links. It seems appropriate to reserve the use of pedestrian links with type 1 links for short transfer links.

7 ALGORITHM IMPLEMENTATION

7.1 Definitions

- $l_{i,j,l}$ link between i and j, line l
- $t_{i,j,l}$ travel time on link i,j,l
- $T_{i,j,l}$ average travel time on link i,j,l
- $m_{i,j,l}$ minimum minimum time on link i,j,l
- $M'_{i,j,l}$ minimum maximum time on link for non infinite lines i,j,l
- $M_{i,j,l}$ minimum maximum time on link i,j,l including possible infinite frequency strategy
- $hdw_{i,j,l}$ headway of the line segment i,j,l , 0 if walk link

- $V_{i,j,l}$ link volume i,j,l
- $S_{i,j,l}$ set of strategies $s_{i,j,l,k}$ of link i,j,l
- $\mu_{i,j,k,l}$ minimum time for the k strategy, link i,j,l
- $\nu_{i,j,k,l}$ maximum time for the k strategy, link i,j,l
- $n_{i,j,k,l}$ hourly frequency of k strategy, link i,j,l
- $h_{i,j,k,l}$ headway of k strategy , link i,j,l
- $p_{i,j,k,l}$ proportion of demand, assigned on k strategy, link i,j,l
- $\pi_{i,j,k,l}$ previous link from k strategy, link i,j,l
- Φ set of non reached links
- P set of reached links
- Ω set of optimized links
- O trip origin
- D destination trip
- In() : set of predecessors
- Out() : set of successors

7.2 Principle

We start from the destination to determine attractive strategies

- Step 0 : Initialization

$P=\mathrm{In}(D)$

$\forall\, l_{i,j,l} \in P$, we add a new k strategy, defined by :

- $T_{i,j,l} = M_{i,j,l} = m_{i,j,l} = \mu_{i,j,k,l} = \nu_{i,j,k,l} = t_{i,j,l}$
- « l » transit line identifier

7.3 Step 1: Main loop

- *Step 1.1 :Calculation and link labeling*

We take $l_{i,j,l} \in P$ with minimum $T_{i,j,l}$ as pivot

$P=P-\{l_{i,j,l}\}\ et\ \Omega=\Omega\cup\{l_{i,j,l}\}$

For each predecessor u in $l_{i,j,l}$:

- if u has not been reached yet $u \notin P$
  - if u and $l_{i,j,l}$ belong to different lines, each with a non infinite frequency
    - We build the average strategy s from the set of strategies of

$l_{i,j,l}$

- $S_u = s$
- s is defined by:

$\mu_s = T_{i,j,l} + t_u$
$\nu_s = T_{i,j,l} + t_u + hdw_{i,j,l}$
$p_s = 1$
$\pi_s = l_{i,j,l}$

- else, the u set of strategies is the $l_{i,j,l}$ set of strategies where the travel time of u will be add to the minimum, average and maximum times. The distribution of flow according to the strategies remains the same .
  - $S_u = S_{i,j,l}$
  - $\mu_{u,k} = \mu_{i,j,k,l} + t_u$
  - $\nu_{u,k} = \nu_{i,j,k,l} + t_u$
  - $m_u = m_{i,j,k} + t_u$
  - $M_u = M_{i,j,l} + t_u$
  - $T_u = T_{i,j,l} + t_u$
  - $M'_u = M'_{i,j,l} + t_u$
  - $\forall\ k \in S_{u,k},\ p_u = p_{i,j,k,l}$

- if u has been reached ($u \in P, \Rightarrow S_u \neq \emptyset$)
  - If u and $l_{i,j,l}$ belong to different transit lines, where both headway are not zero $m_{i,j,l} + t_u < M_u$, we can optimize
    - We build the average strategy “s” from the $l_{i,j,l}$ set of strategies

$\mu_s = T_{i,j,l} + t_u$
$\nu_s = T_{i,j,l} + t_u + hdw_{i,j,l}$
$p_s = 1$
$\pi_s = l_{i,j,l}$

    - If $s \in S_u$ and as $T_{i,j,l} + t_u < T_u$, we update s strategy from u, which become optimized, else we add “s” to the u set of strategies $S_u = S_u \cup \{s\}$
  - Else if $m_{i,j,l} + t_u < M_u$ then we can optimize
    - $\forall\ s \in S_u \cap S_{i,j,l}$, if $\mu_s + t_u < T_u$, then we can update s strategy from u , which become optimized, else we keep existing values
    - After,

$\forall\ s \notin S_u et\ s \in S_{i,j,l}, S_u = S_u \cup \{s\} si\ \mu_s + t_u < T_u et\ hdw_s > 0\ ou\ \mu_s + t_u < M_u si\ hdw_s = 0$
we add to u set of strategies, the $l_{i,j,l}$ strategies which are not belonging to u and enable an optimization of u.

- *Step 1.2: M and M' updates*

We update m, M and M' values from u from its own strategies

- $m_u = Min(\mu_k), k \in S_u$
- $M'_u = Min(\mu_k + h_k) \forall\ k \in S_u$
- $M_u = M'_u si \forall\ k \in S_u, n_k > 0$
  $else$
  $M_u = Min(\mu_k) \forall\ k \in S_u tel\ que\ n_k = 0$
- $\delta = \frac{(\sum_{s \in S_u} n_s(M - \mu_s)) - 60}{\sum_{s \in S_u} n_s}$
- $M' = M' - \delta$
- $' = Min\{M\ , M\ '\}$

- *Step 1.3: Strategies weights update*
- $\forall\ k \in S_u$
  - If $n_k > 0$, $p_{u,k} = \frac{M - \mu_{u,k}}{hdwy_{u,k}}$
  - If $n_k = 0$, $p_{u,k} = (M' - M)(\sum_{k \in S_u} \frac{1}{hdw_{u,k}}) = 1 - \sum_{k \in S} \frac{M - \mu_{u,k}}{hdwy_{u,k}}$
  - If $\sum_{k \in S_u} n_{u,k} = 0$, $p_{u,k} = 1$
- *Step 1.4: Expected travel time update*
- $T_u = \frac{1}{2} \sum_{k \in S_u} p_{u,k}(\mu_{u,k} + M)\ \forall\ k \in S_u$ if $\sum_{k \in S_u} n_{u,k} > 0$
- $T_u = M$ si $\sum_{k \in S_u} n_{u,k} = 0$

7.4 Step 2: Allocation of demand in proportion to the weight of the strategies

We start from origin,

- *Step 2.1 Initialization*

$P = \mathrm{Out}(O)$

- *Step 2.2 Main loop*

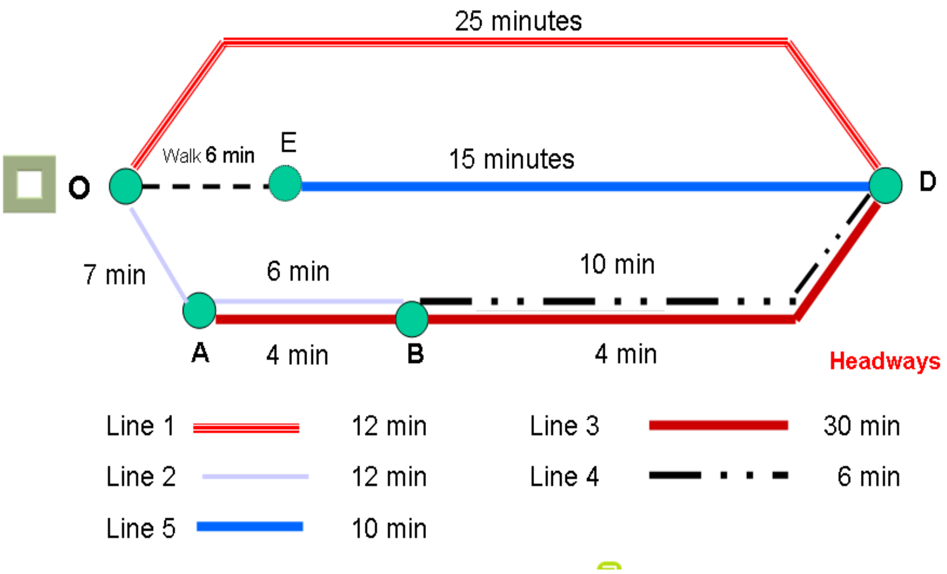
$\forall\ u \in P, \forall\ k \in S_u, V_{\pi_u} = V_{\pi_u} + V_u \cdot p_{u,k}$

if π_u exists, we add π_u to P, $P = P \cup \{\pi_u\}$ and we drop u from P, $P = P - \{u\}$

Repeat until $P = \emptyset$

8 APPLICATION ON A TEST NETWORK

This test network, taken from the Emme user manual; has been used in particular by INRO to show the improvements of the transit assignment with variant module, recently implemented in Emme. This network is a good test case for comparing the Mint results with optimal strategies with and without variants.



8.1 Optimal strategies result

The optimal strategy results in the use of a single alternative: 6min walk form O to E and then line 5.

T= 26.0 minutes

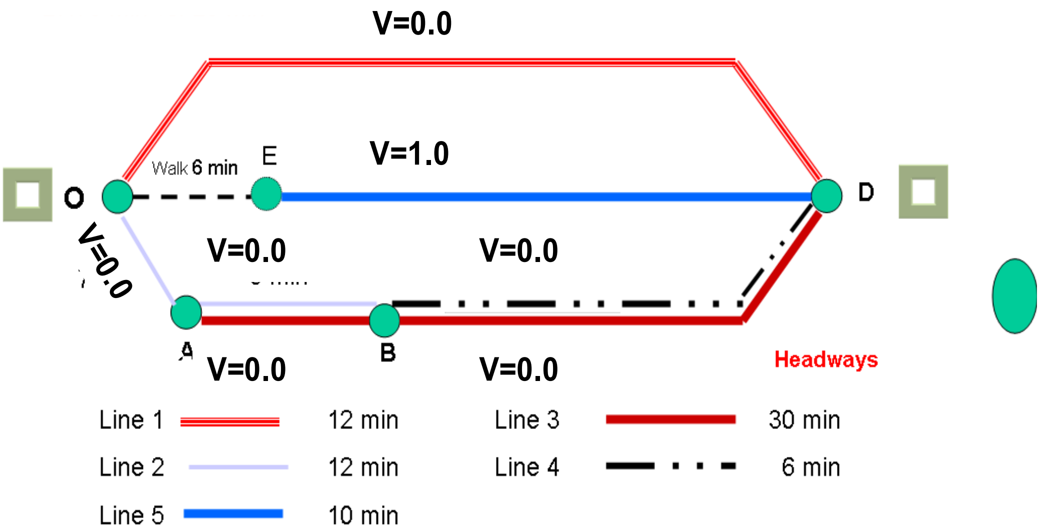


Figure 10: Distribution of flows with the method of optimal strategies.

8.2 Optimal strategies with variants

With the logit choice method, all options are used, except line 3 which would imply an alighting before the end of line 2.

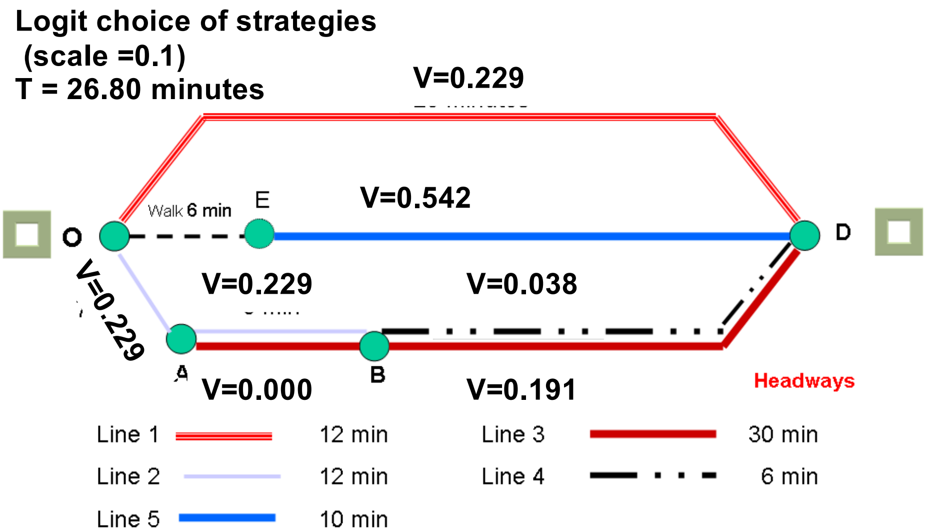


Figure 11: Distribution of flows with the method of optimal strategies with variants

8.3 Mint results

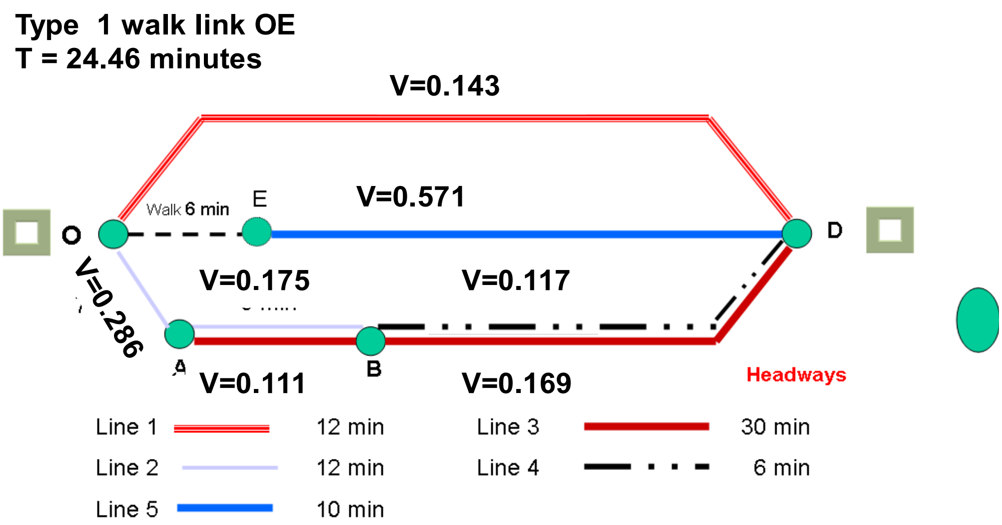


Figure 12: Distribution of flows with the Mint method and a walk link of type 1

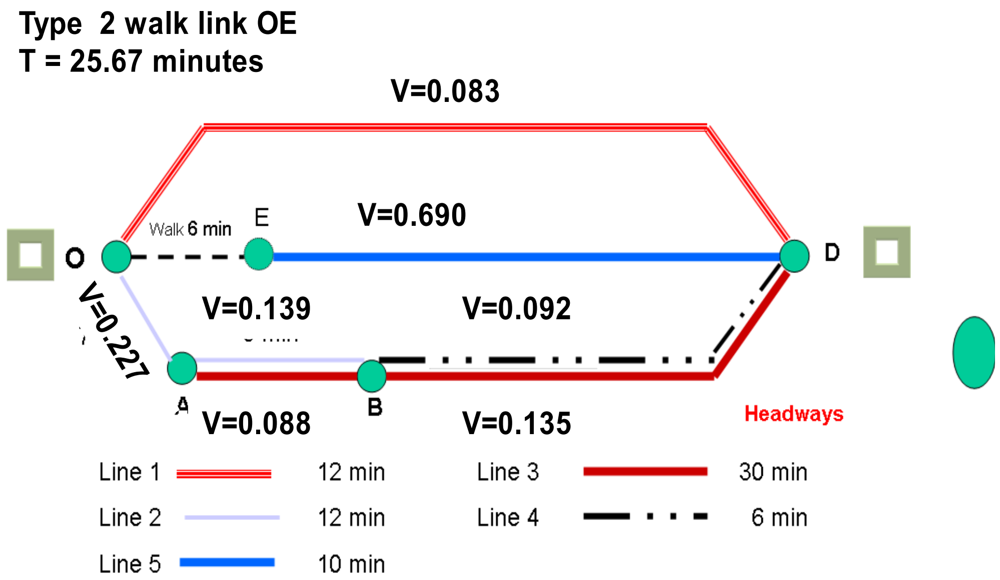


Figure 13: Distribution of flows with the Mint method and a walk link of type 2

8.4 Mint results in generalized time

Walking weight : 2

Wait weight : 2

Boarding time :2

Boarding weight : 2

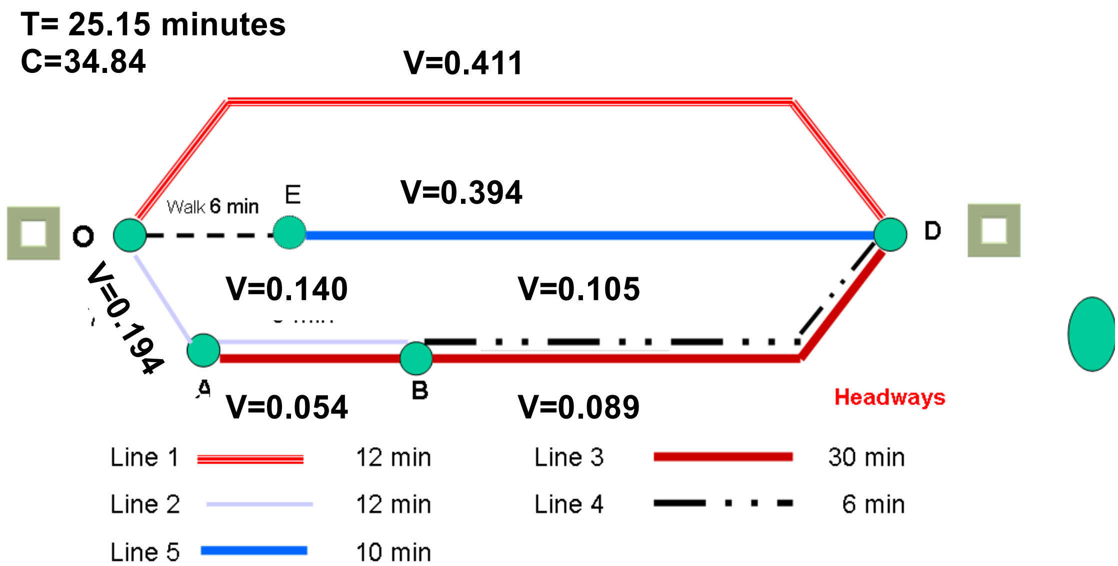


Figure 14: Distribution of flows with the Mint method in generalized time and a walk link of type 2

Unlike optimal strategies, even if the generalized time does not change the set of attractive lines, the distribution of flows is changed according to the different weights of the strategies.

8.5 Test on Winnipeg demo bank

This test was performed on the Emme demo bank on Winnipeg.
The network contains 154 zones, 900 nodes and 3000 links. The main interest is to analyze the Mint behaviour on a real network and not just on trivial cases. The results below, seem consistent compared to those of Emme

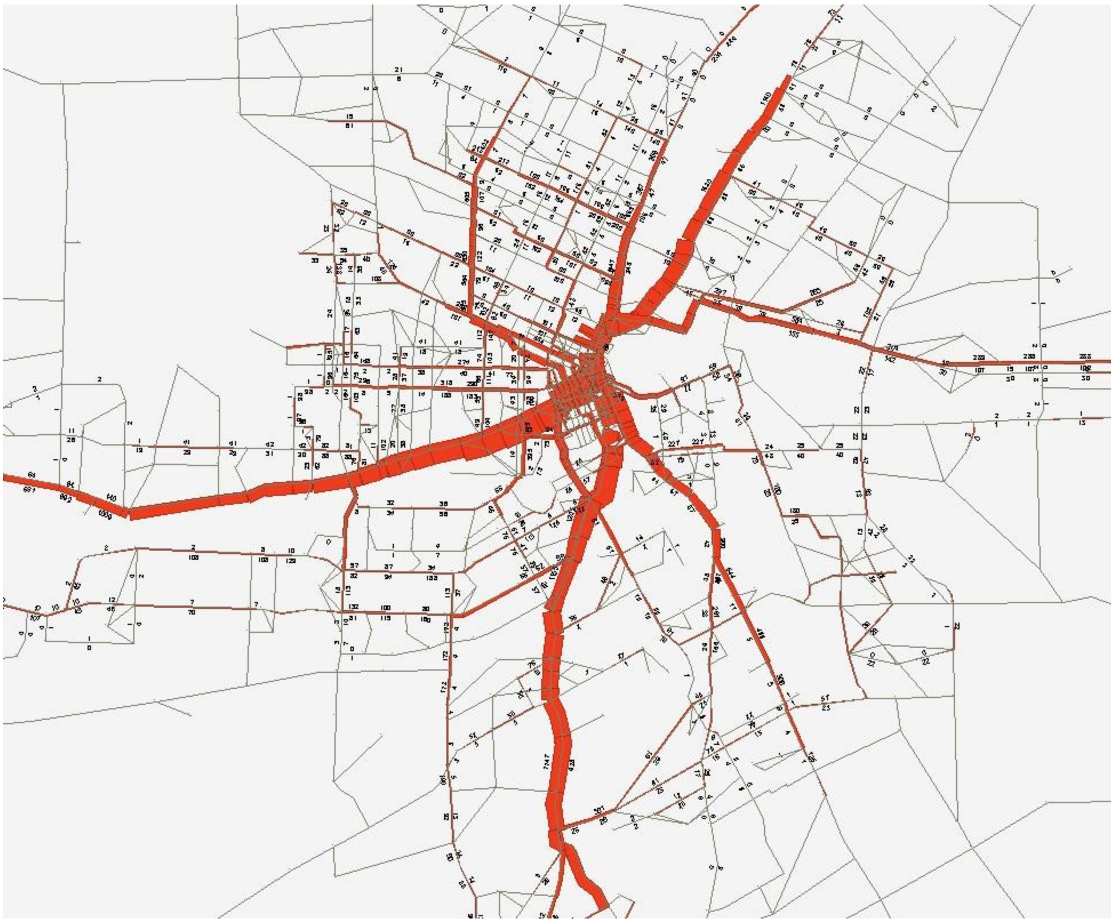


Figure 15: Winnipeg demo Mint transit assignment

9 TEST MINT ON THE PARIS URBAN AREA

The objective of the test is to evaluate the Mint behavior on large networks.

Paris network size :

- 1293 zones
- 17404 regular nodes
- 60358 links

- 4577 transit lines
- 71279 transit segments

The assignment has performed well ans has taken 4h50 of computing.

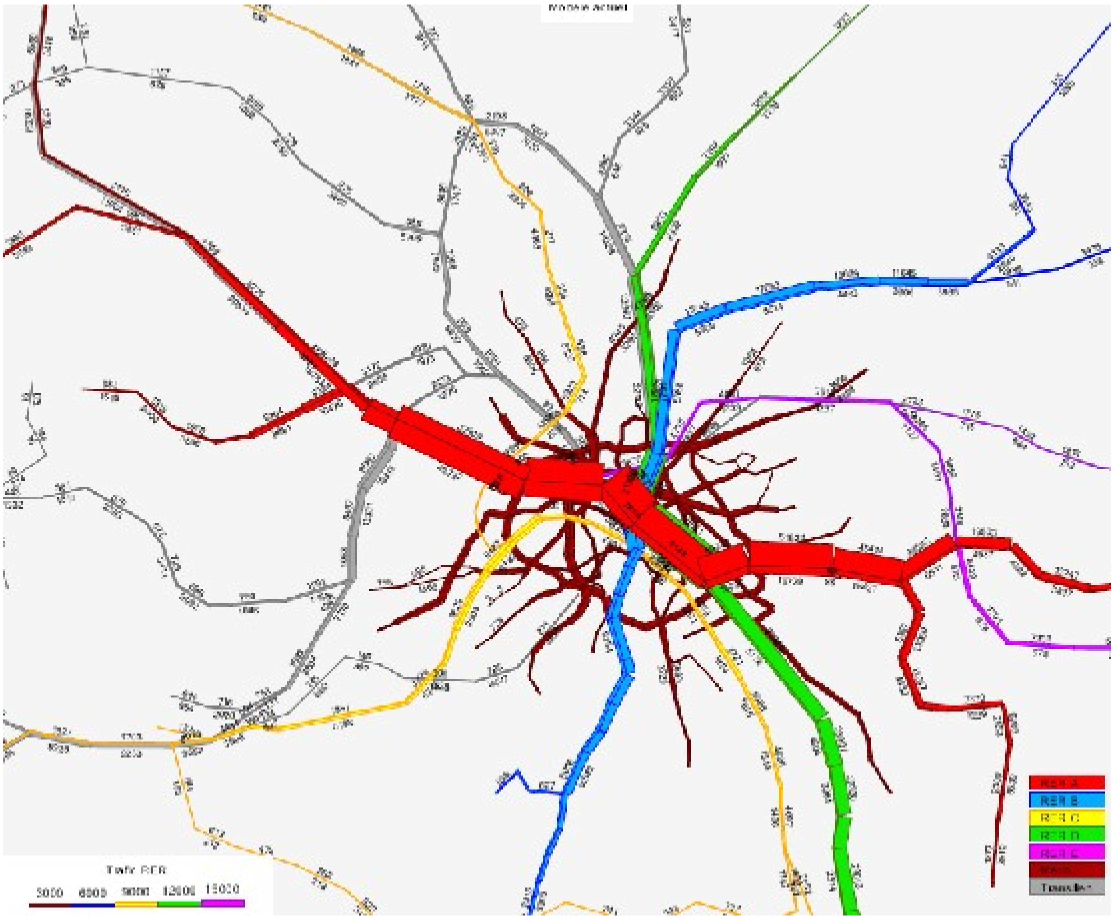
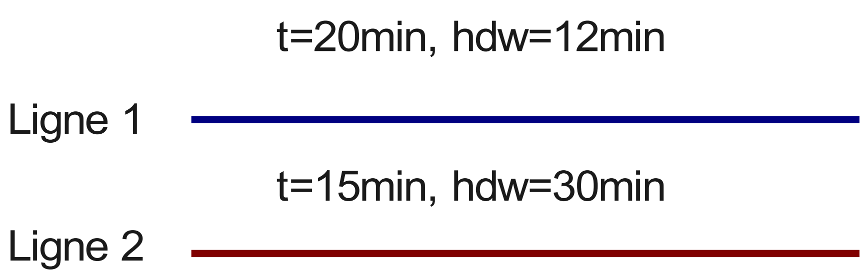


Figure 16: Mint assignment on Paris area transit network (rail lines view)

10 IMPACT ON TIMETABLES

We take the example 2 below, which consists of two lines with different frequency and different travel times.



First, we suppose that both headways are regular. We obtain the distribution of time between A and B for each of the two lines depending on the departure time

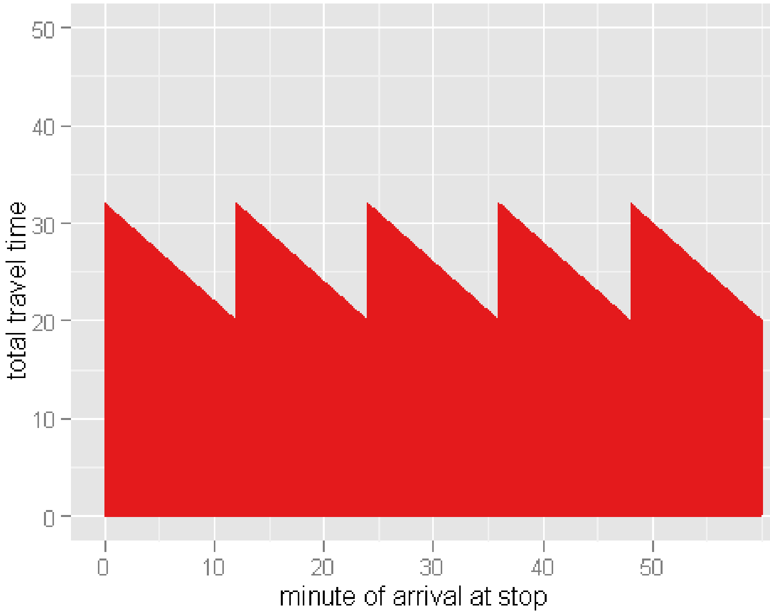


Figure 17: Line 1 travel time distribution

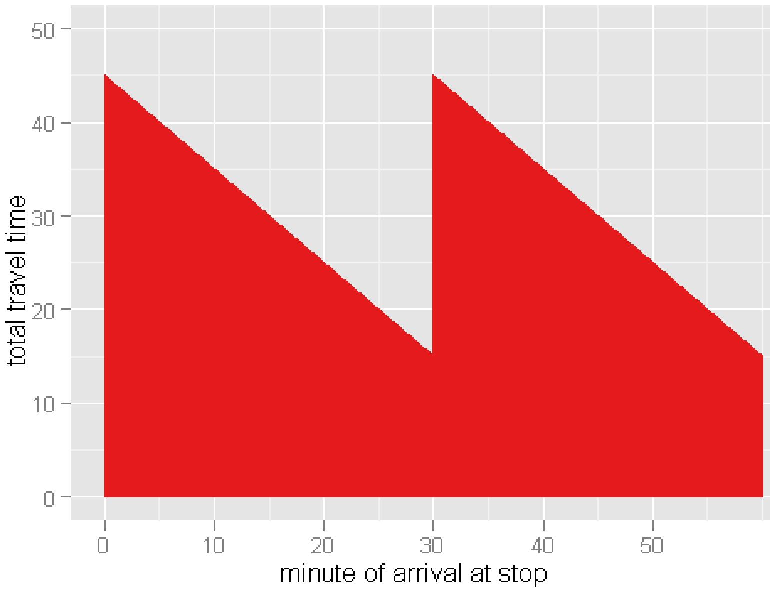


Figure 18: Line 2 travel time distribution

As both lines are attractive, the user will have the choice to use either line 1 or line 2.
This is equivalent to using a line, combination of lines 1 and 2, defined by the combined frequency of both lines

The expected travel time depends on how both lines are listed in timetables

10.1 Regular inter-vehicular interval on each line

Initially, we consider that the inter-vehicular intervals of both lines are regular and timetables are defined as follows:
Line 1 : 0,12,24,36,48
Line 2: x,30+x where x is varying from 0 to 30

The impact of timetable on the expected travel time will be analyzed according to two strategies

- The user boards into the first vehicle that comes to him (Optimal Strategies)
- The user boards into the vehicle that minimizes travel time and waiting time (Mint). In this strategy, the user does not board into the first vehicle if another arriving after enable him to reach its destination sooner.

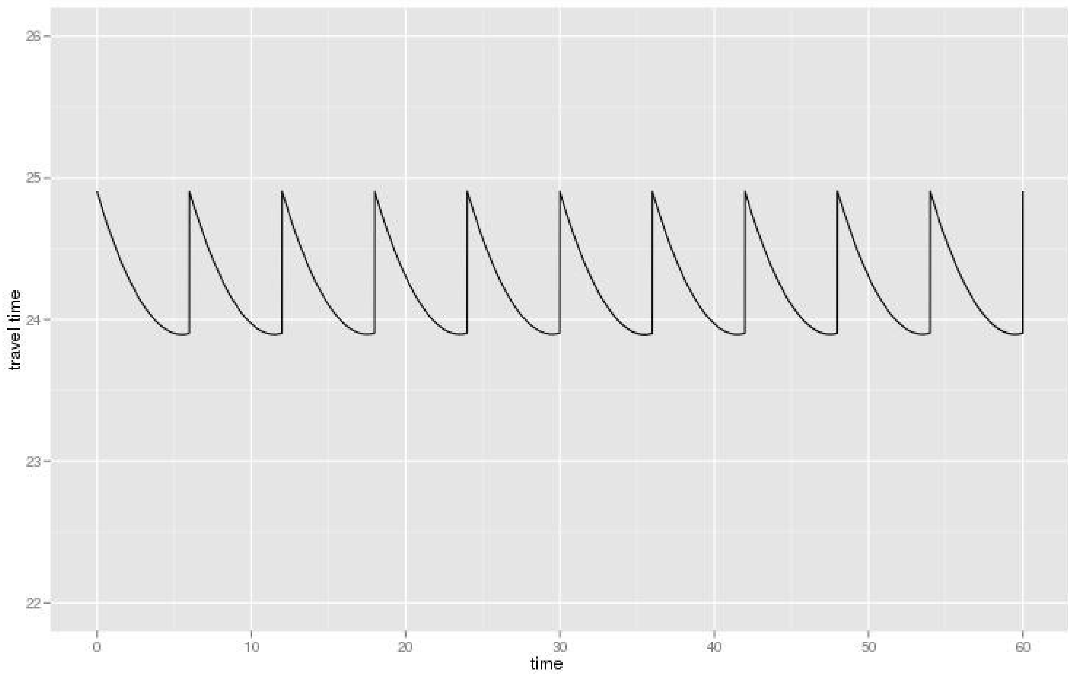


Figure 19: First vehicle boarding strategy

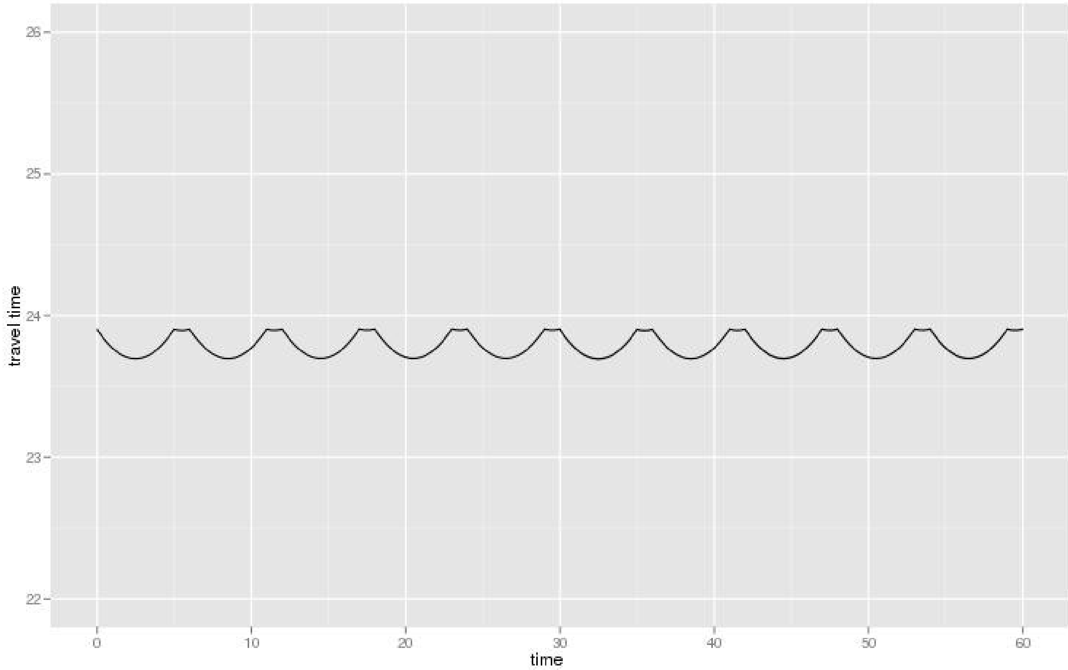


Figure 20: Minimum maximum time strategy

The two diagrams show that if the inter-vehicular intervals are uniform for each line, the expected travel time, combining both lines is always higher than the one determined by either optimal strategies or Mint.

Results of the simulation are:

- First vehicle boarding strategy: Min:23.89 min Max : 24.91 min Mean:24.21 min
- Maximum minimum time strategy : Min : 23.69 min Max : 23.91 min Mean:23.79 min

As a reminder, the values found by both algorithms are:
Optimal strategy travel time: 22.86 minutes
Mint travel time: 22.56 minutes

These results show that the timetables that are implicitly used by optimal strategies and Mint are not those taken earlier, and therefore that the inter-vehicular intervals are not, for this example, uniform for each line.
Therefore, both methods are based on implicit timetables, results of an optimization

10.2 Random offset

We still suppose that timetables for each line are regular. Over all sets of timetables an arbitrary passenger will face impedance given by:

$T_1 = 20 + 12.\alpha_1$
$T_2 = 15 + 30.\alpha_2$
α_i *are random variables on the unit interval*

We can make 10000 draws of the pair (α_1, α_2) and for each draw taking min(T1, T2) as the choice of line.

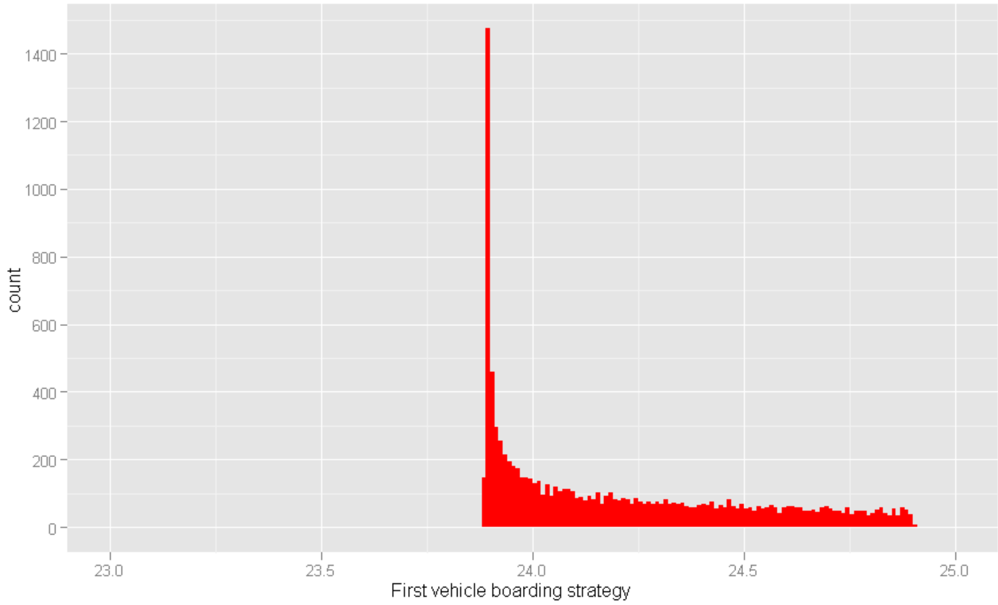


Figure 21: Result of the 10000 draws of α_1, α_2 *, first vehicle boarding strategy*

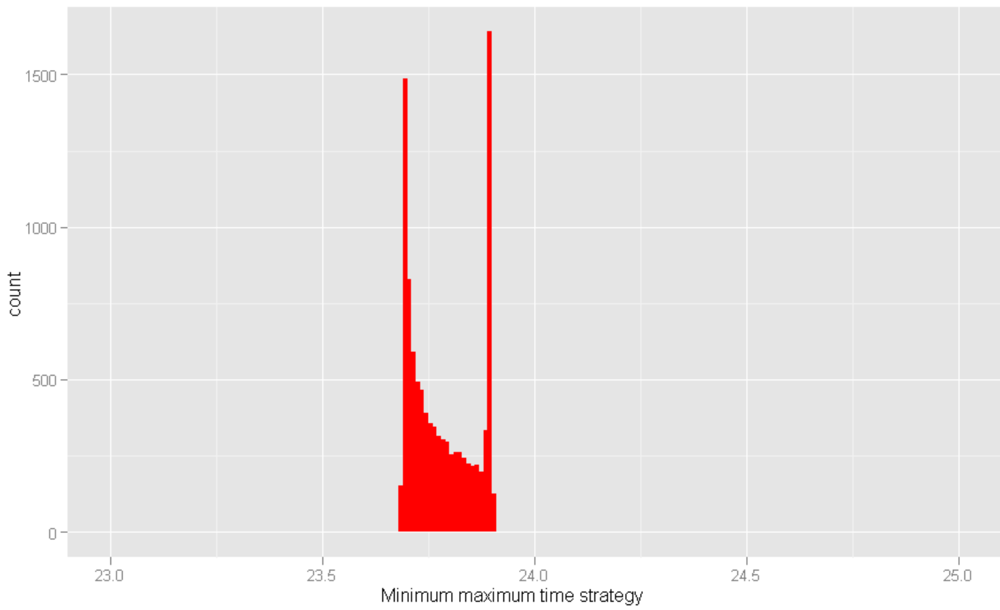


Figure 22: Result of the 10000 draws of α_1, α_2 *, minimum maximum boarding strategy*

	First vehicle boarding	Minimum maximum time
Minimum:	23,89	23,69
Maximum:	24,90	23,90
Mean	24,20	23,78
Standard deviation:	0,30	0,075

For regular timetables and random offsets for both lines, travel times are greater than optimal strategies of Mint travel times. Minimum maximum time strategy is always quicker and the expected travel time has very low variations depending of both line offsets

10.3 Completely random timetables

The objective is to study, with constant frequency and travel time for each, the impact on the structure of the timetables and the expected travel time.

The method consists in analyzing the distribution of expected travel times, calculated for a large number of randomly generated timetables for each line. The only restriction on timetables is to require for each line a number of passes equal to their hourly rate.

For this, we perform Monte-Carlo simulations by generating 10,000 timetables for lines 1 and 2, and estimate the expected travel time

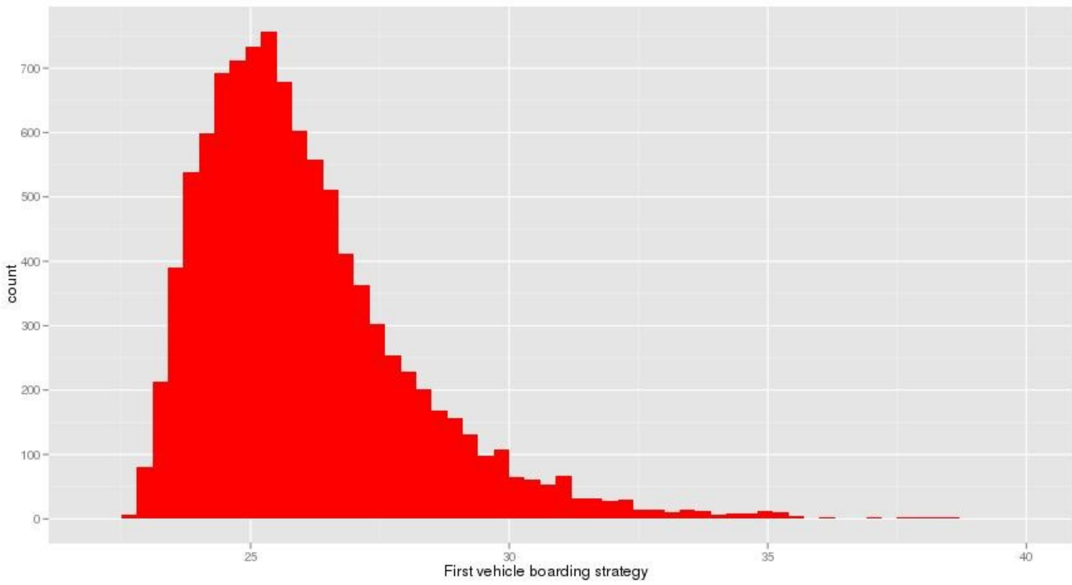


Figure 23: Distribution of average travel time for random timetables on line 1 and line 2 with first boarding strategy

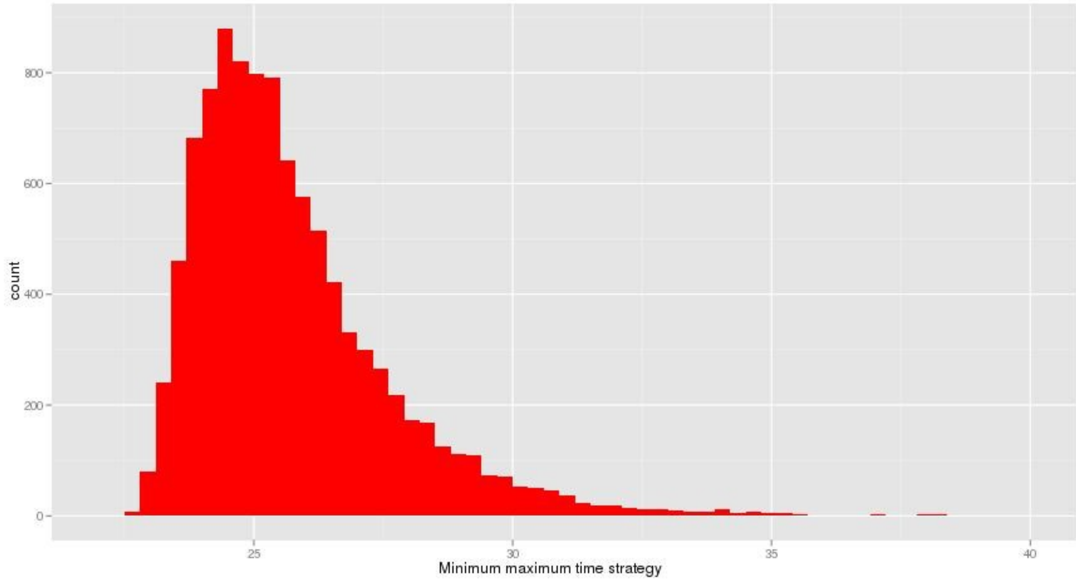


Figure 24: Distribution of average travel time for random timetables on line 1 and line 2 with minimum maximum time strategy

Simulation results are:

	First vehicle boarding	Minimum maximum time
Minimum:	22,63	22,63
Maximum:	38,83	39,52
Mean	25,74	26,07
Standard deviation:	1,91	2,09

These results show that the average expected travel time is greater than the overall average travel time determined by the two approaches (optimal strategies and Mint). In addition, we find, for several schedules, that the expected travel time is lower than the one determined by the optimal strategies.

The Mint expected travel time seems to be the optimum. We will see hereafter that

- it is based on a timetable built from the concept of equalilty of the maximum time.
- a schedule of equal maximum time is optimal, by showing that any other schedule has an expected travel times higher than Mint.

10.4 Optimal strategies implicit timetables

The method of optimal strategies is based on the fact that the inter-vehicle intervals from the combination of attractive lines is uniform. The timetable can be built by calculating the number of vehicle of both lines, and determining the average interval, which will be assumed regular

In our example, there are 5 vehicle arrivals per hour for line 1 and 2 for line 2 The combined frequency is 7 vehicles per hour regularly spaced by an interval of 60 / (5 +2) = 8.57 minutes

MINUTES	LINE	TRAVEL_TIME
0.00	1	20
8.57	1	20
17.14	1	20
25.71	2	15
34.29	1	20
42.86	1	20
51.43	2	15

Table 1: Optimal strategies timetable

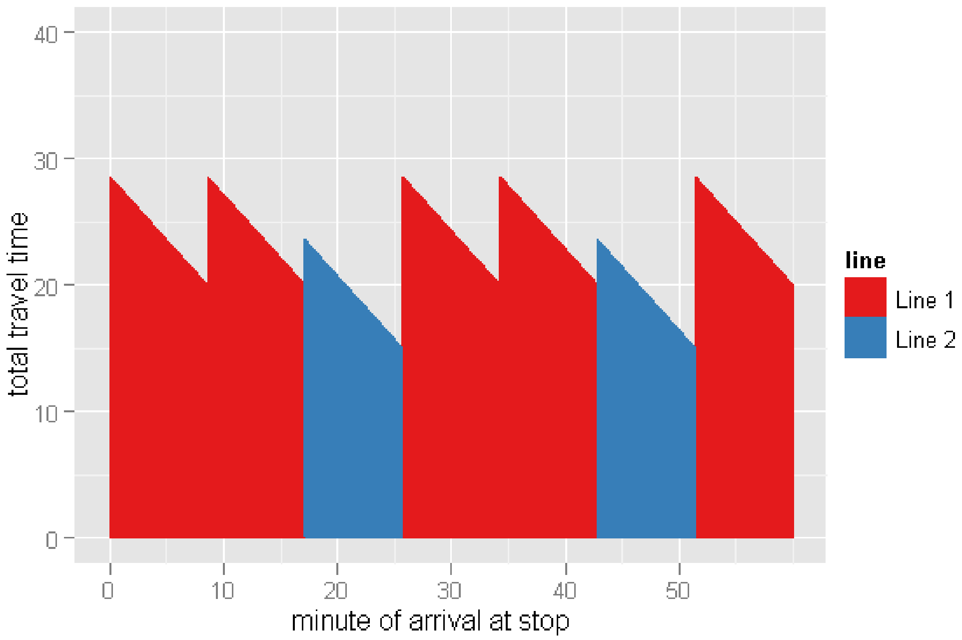


Figure 25: Optimal strategies implicit timetable

10.5 Mint implicit timetables

Mint is based on an optimization of the concept of maximum time. The maximum time is defined as the sum of travel time and the interval between two consecutive vehicles. When travel times of both lines are identical, the implicit timetables of optimal strategies and Mint are identical; The expected travel time of both methods is then identical and is optimal, if these only two lines are attractive.

If travel times on lines 1 and 2 are different, the expected travel time determined by the method of optimal strategies is not minimum. The optimal expected travel time is given by Mint, which minimizes the maximum time.

We can show that the optimal solution is given by Mint, with a associate timetable based on the principle of equal maximum time

Suppose we have a Mint timetable (diagram below). The expected travel time is given by the ratio of the area under the curve and the duration of the period (T = 60 minutes in our case), and for an inter-vehicular interval, we apply a positive or negative shift δ_t

Δ_t is the difference on the total expected travel time between the two timetables

$$\Delta_T = \frac{(M.\delta_t + \frac{\delta_t^2}{2} - ((M-\delta_t).\delta_t + \frac{\delta_t^2}{2}))}{T}$$

$$\Rightarrow \Delta_T = \frac{\delta_t^2}{T}$$

Δ_t is always positive, and therefore, whatever the changes applied to the Mint timetable, the expected travel time of Mint will always be inferior.

MINUTES	LINE	TRAVEL TIME
0.00	1	20
7.14	1	20
14.29	1	20
26.43	2	15
33.57	1	20
40.71	1	20
52.86	2	15

Tableau 2: Mint timetable

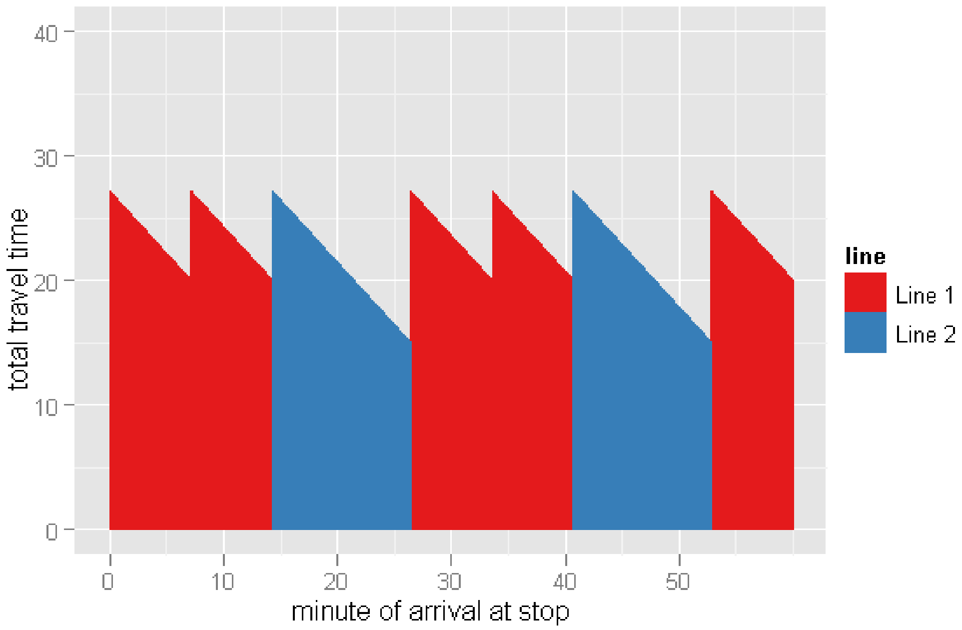


Figure 26: Mint implicit timetable

10.6 Timetable with walking strategy

With the example above, we can visualize the impact of a walking strategy on the timetable, which consists in adding a walking strategy of 25minutes.

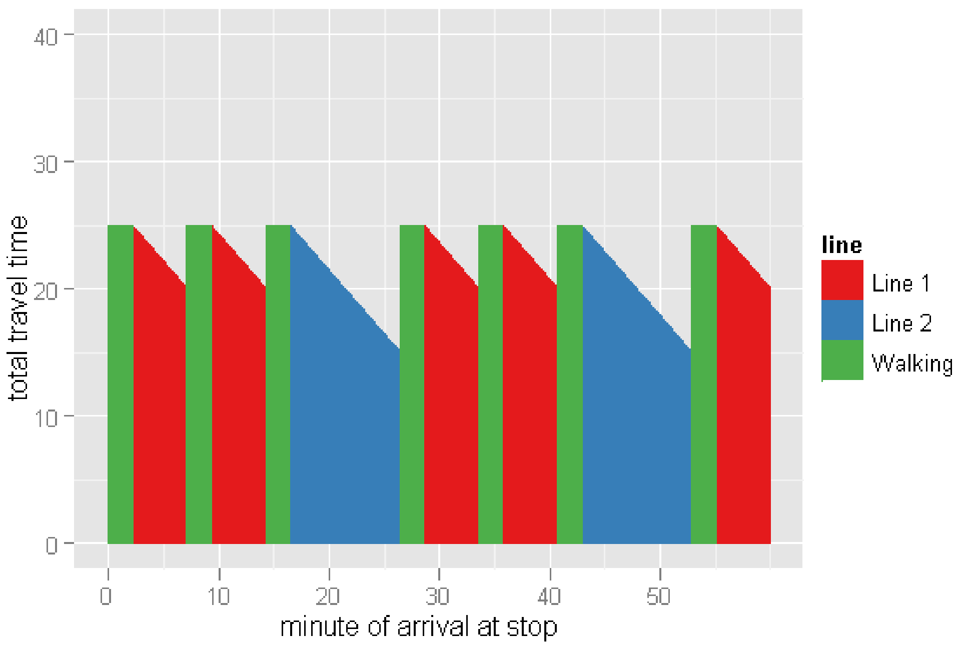


Figure 27: Mint implicit timetable with walk only strategy

11 TECHNICAL CONSIDERATIONS FOR THE OPERATIONAL IMPLEMENTATION OF MINT

11.1 The Mint tool

CETE Nord-Picardie has developed a software "Mint", which implements the eponymous algorithm.

The tool developed in C # works with

- a transit network codified by frequencies
- a demand matrix
- assignment parameters: different weights for the generalized cost
- an algorithm for optimizing node labelling

Network and matrices files are text files with separator ";"

The network is described by a list of line segments. A segment is defined by an initial node identifier, an end-node identifier, a transit line identifier. Walking links are considered as line segments with an infinite frequency (headway = 0).

For each segment, we must complete the following fields:

- initial node identifier (text)
- end node identifier (text)
- transit line identifier (text)
- travel time (real)
- headway (real)
- vehicle total capacity (real)
- allow boarding on initial node (0 = no, 1 = yes)
- allow alighting on end node (0 = no, 1 = yes)

The demand matrix file is defined by a list of origin-destination associated with a corresponding volume:

- origin node identifier (text)
- destination node identifier (text)
- volume of demand on the origin destination (real)

Mint produces three output files:

- The results of assignment for each segment: volume, number of boardings, number of alightings

- Travel times matrix: expected travel time , expected generalized time; average transfer ratio
- An optional strategies file, which describes for each segment, all level one attractive strategies, with indications of percentage of volume per strategy, the maximum minimum time

11.2 Generalized time

The algorithm is described here, regardless of the various weights of different components of time, which changes slightly the formulas but not the principle.

In the current prototype version of Mint, the following parameters are considered:

- walking weight:
- wait weight:
- boarding time:
- boarding weight:

11.3 Consideration of walking strategies

The proportion of volume, for walking strategies are given by the following formulas.

$$p_{u,k}=(M-M')\frac{(\sum_{k\in S_u} n_{u,k})}{60}=1-\sum_{k\in S}\frac{M-\mu_{u,k}}{hdwy_{u,k}}$$

Walking links are equivalent to transit lines with infinite frequency. If we can not apply the same formulas as for the classic lines, it is possible to demonstrate the above formulas by considering instead of the pedestrian link, a line with an inter-vehicular interval ϵ which tends to 0. The proportion of the walking strategy is obtained by:

$$\lim_{\epsilon\to 0} p_{u,k}(\epsilon)=1-\sum_{k\in S}\frac{M-\mu_{u,k}}{hdwy_{u,k}}$$

11.4 Independence of strategies

During the operational implementation of Mint, it is essential to ensure the independence of strategies. For this, we must ensure, in combining strategies, not to add the same line twice, which would have the effect of reducing the maximum minimum in an artificial and wrong way.

11.5 Problem of cycles

For a correct functioning of the algorithm, it is necessary to avoid the possibility of cycles in the search of attractive strategies, which can artificially enhance the overall travel time incorrectly and prevent the convergence. To avoid such a malfunction, a line segment or pedestrian link can be added, only if it doesn't belong to set of segments and links constituting all existing strategies.

11.6 Computing optimization

In the operational implementation of Mint, several costs are involved in the optimization, making algorithms commonly used, such as Dijsktra not directly usable, because the time of the next segment may be lower than the pivot one, if it is located after the combination of several lines.

The algorithm used in the Mint software is the "graph growth algorithm with buckets. The size of the intervals varies and depends on the square root of the travel time, to try to store a constant number of segment in each interval. The computing time is longer than the optimal strategies process, since the assignment procedure on a Emme Winnipeg demo bank takes a few minutes versus a few seconds. However, computation time of the current algorithm could be improved with a more efficient implementation.

11.7 Consideration of capacity constraints

In Mint, the flow distribution is a continuous function of frequency and travel time. This property may enhance the convergence of an iterative algorithm taking into account capacity constraints.

It is also possible that this algorithm can allow the implementation of a more efficient convergence process than the method of successive average currently used in the macro captras implemented in Emme. Similarly, the properties of Mint can bear other procedures to take into account capacity constraints. For example specific boarding time delays can be used instead of effective headways, essential with optimal strategies.

12 CONCLUSION

In conclusion, the methods of optimal strategies and Mint are based on strong assumptions and comparable information on the structure of their associated timetables. The only difference between the two methods is how to build these implicit timetables.

Mint overcomes most of the limitations of optimal strategies and provide the optimal expected travel for a set of attractive lines.

For optimal strategies, the inter-vehicular interval is supposed constant, whereas the method Mint, this is the maximum time

The Mint algorithm determines expected travel time for a given set of attractive lines, and has the advantage of providing a distribution of flows on lines that depend on the respective travel time.

The current version of Mint was tested on the two networks of different sizes, respectively Winnipeg and Paris. The results are encouraging and the algorithm seems to perform well.

13 BIBLIOGRAPHY

Florian M, Constantin I.(2011), *Emme Strategy Transit Assignments with Variants*, INRO, Montréal, QC, Canada, Ontario Emme Users' Group / ITE Toronto Meeting

INRO (2011), Emme prompt manual, INRO, Montréal, QC, Canada

R Development Core Team (2011). *R: A language and environment for statistical computing. R Foundation for Statistical Computing*, Vienna, Austria.

Spiess H., Florian M. (1989), Optimal strategies: A new assignment model for transit networks, *Transportation Research Part B: Methodological*, **23** (2) 83-102

Wickham H. (2009), *ggplot2: elegant graphics for data analysis*, Springer New York,